

FRACTIONAL LYAPUNOV INEQUALITIES ON SPHERICAL SHELLS

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Abstract. This paper, deals with Lyapunov inequalities of conformable fractional boundary value problems on an N -dimensional spherical shell. Applicability of these Lyapunov inequalities will be examined by establishing the disconjugacy as a nonexistence criterion for nontrivial solutions, lower bound estimation for eigenvalues of the corresponding fractional eigenvalue problem, upper bound estimation for maximum number of zeros of the nontrivial solutions and distance between consecutive zeros of an oscillatory solution.

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