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MONOTONE DYNAMICS OR NOT? DYNAMICAL CONSEQUENCES OF VARIOUS MECHANISMS FOR DELAYED LOGISTIC GROWTH

TORSTEN LINDSTRÖM

Abstract. In this paper we interpret the global stability properties of the delayed single species chemostat in terms of monotone dynamics on an asymptotically invariant hyperplane in the state space. The consequence is a translation of advanced analysis and delay differential equations into sign checks and ordinary differential equations for an important single species model with explicit resource dynamics. Complete proofs are included, since the limiting behavior at asymptotically invariant sets may not agree with the limiting behavior of the original system even in the finite dimensional case (Thieme (1992)).

A delayed logistic equation based on explicit resource dynamics falls out as a limiting case of the chemostat and we claim this to be a new mechanistic interpretation of delayed logistic models. We continue by comparing these results to several other delayed logistic models that has been mechanistically justified in the literature. We conclude that monotone dynamics apply in several cases. We improve one global stability result that cannot be obtained with by the use of monotone dynamics and end up by pointing out the dynamical differences between Hutchinson's (1948) delayed logistic equation and those with mechanistic interpretations.

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dea@ele-math.com