

## TWO-SCALE CONVERGENCE IN THIN DOMAINS WITH LOCALLY PERIODIC RAPIDLY OSCILLATING BOUNDARY

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*Abstract.* The aim of this paper is to adapt the notion of two-scale convergence in  $L^p$  to the case of a measure converging to a singular one. We present a specific case when a thin cylinder with locally periodic rapidly oscillating boundary shrinks to a segment, and the corresponding measure charging the cylinder converges to a one-dimensional Lebesgue measure of an interval. The method is then applied to the asymptotic analysis of linear elliptic operators with locally periodic coefficients and a  $p$ -Laplacian stated in thin cylinders with locally periodic rapidly varying thickness.

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