

## VANISHING MAGNETIC FIELD LIMITS OF SOLUTIONS TO THE PRESSURELESS MAGNETOGASDYNAMICS

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*Abstract.* We are interested in the system of conservation laws modeling the pressureless magnetogasdynamics. Firstly, we solve the Riemann problem and obtain five kinds of structures consisting of combinations of shocks, rarefaction waves and contact discontinuities. Secondly, we study the vanishing magnetic field limits of the Riemann solutions to the pressureless magnetogasdynamics and show that the density and velocity in the Riemann solutions to the pressureless magnetogasdynamics converge to the Riemann solutions to the pressureless gas dynamics. The formation processes of delta-shocks and vacuum states are discussed in details.

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