

ON BOUNDARY VALUE PROBLEM FOR EQUATIONS WITH CUBIC NONLINEARITY AND STEP-WISE COEFFICIENT

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Abstract. The differential equation with cubic nonlinearity $x'' = -ax + bx^3$ is considered together with the boundary conditions $x(-1) = x(1) = 0$. In the autonomous case, $b = \text{const} > 0$, the exact number of solutions for the boundary value problem is given. For nonautonomous case, where $b = \beta(t)$ is a step-wise function, the existence of additional solutions is detected. The reasons for such behaviour are revealed. The example considered in this paper is supplemented by a number of visualizations.

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