

## $L^2$ -CONCENTRATION FOR A COUPLED NONLINEAR SCHRÖDINGER SYSTEM

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*Abstract.* In this work we adapt Bourgain’s ideas in [?] to a coupled system and we prove the  $L^2$ -concentration of blow-up solutions for two-coupled nonlinear Schrödinger equations at critical dimension.

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