

BOUNDS FOR GLOBAL SOLUTIONS OF A REACTION DIFFUSION SYSTEM WITH THE ROBIN BOUNDARY CONDITIONS

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Abstract. In this paper, we are concerned with the large-time behavior of solutions of a reaction diffusion system arising from a nuclear reactor model with the Robin boundary conditions, which consists of two real-valued unknown functions. It is shown that global solutions of this system are uniformly bounded in a suitable norm with respect to time.

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