

MULTIPLE SOLUTIONS FOR A FOURTH ORDER EQUATION WITH NONLINEAR BOUNDARY CONDITIONS: THEORETICAL AND NUMERICAL ASPECTS

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Abstract. We consider in this work the fourth order equation with nonlinear boundary conditions. We present the result for the existence of multiple solutions based on the Avery-Peterson fixed-point theorem. This work is also a study for numerical solutions based on the Levenberg-Maquardt method with a heuristic strategy for initial points that proposes to numerically determine multiple solutions to the problem addressed.

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