

A NON GREEN'S FUNCTION APPROACH TO FRACTIONAL LYAPUNOV-TYPE INEQUALITIES WITH APPLICATIONS TO MULTIVARIATE DOMAINS

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Abstract. We derive Lyapunov-type inequalities for certain fractional differential equations of order α , where $1 < \alpha \leq 2$ or $2 < \alpha \leq 3$. The methods used within rely on considering the maximum value of a nontrivial solution in a given interval as opposed to traditional methods which utilize the Green's function. This particular method provides versatility and can be applied to other fractional boundary value problems where the Green's function is inaccessible. Furthermore, we demonstrate how the inequalities may be extended to fractional multivariate equations in both the left and right-fractional cases.

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