

## POSITIVE SOLUTIONS FOR A FOURTH ORDER DIFFERENTIAL INCLUSION BASED ON THE EULER–BERNOULLI EQUATION FOR A CANTILEVER BEAM

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*Abstract.* An existence result for positive solutions to a fourth order differential inclusion with boundary values is given. This is accomplished by using a fixed point theorem on cones for multivalued maps,  $L^1$  selections and a generalization of the Ascoli theorem. The inclusion allows the function and its first three derivatives to be on the right-hand side. The proof involves a Green's function and a positive eigenvalue of a particular operator. An example is provided.

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