

ASYMPTOTIC PROPERTIES OF SOLUTIONS OF A LANCHESTER–TYPE MODEL

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Abstract. An ordinary differential system referred to as Lanchester-type model is studied. Asymptotic properties of solutions for such systems are considered. In particular, we examine how the limit of the solution as time tends to the infinity varies according to the initial data and we find asymptotic form of solutions that decay to $(0, 0)$.

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