

OPTIMAL CONTROL TO A FACULTATIVE MUTUALISTIC MODEL WITH HARVESTING

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Abstract. In this article, we propose a general facultative mutualistic model with harvesting and investigate an associated optimal control problem. The sufficient and the necessary conditions for the existence of the optimal control are studied. Numerical simulations are carried out to show the efficiency of the proposed control.

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