

EXISTENCE RESULTS FOR SOLUTIONS TO DISCONTINUOUS DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this paper, we present three results about the existence of solutions to discontinuous dynamic equations on time scales. The existence of Carathéodory type solution is produced using convergence and Arzela–Ascoli theorem. The Banach’s fixed point theorem is used to investigate the existence and uniqueness of solutions and using Schaefer’s fixed point theorem we establish the existence of at least one solution. Our results generalizes and extends some existing theorems in this field.

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