Differential Equations & Applications
Volume 12, Number 2 (2020), 165–184 doi:10.7153/dea-2020-12-11

# MULTIPLE SOLUTIONS OF SYSTEMS INVOLVING FRACTIONAL KIRCHHOFF–TYPE EQUATIONS WITH CRITICAL GROWTH

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*Abstract.* In this paper we are going to study existence and multiplicity of solutions of a system involving fractional Kirchhoff-type and critical growth of form

$$\begin{cases} M_1(||u||_X^2)(-\Delta)^s u = \lambda f(x,v(x))\left[\int_\Omega F(x,v(x))dx\right]^{r_1} + |u|^{2_s^*-2}u \text{ in } \Omega, \\ M_2(||v||_X^2)(-\Delta)^s v = \gamma g(x,u(x))\left[\int_\Omega G(x,u(x))dx\right]^{r_2} + |v|^{2_s^*-2}v \text{ in } \Omega, \\ u = v = 0 \text{ in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where $s \in (0,1)$, $n > 2s$, $\Omega \subset \mathbb{R}^n$ is a bounded and open set, $2_s^* = 2n/(n-2s)$ denotes the fractional critical Sobolev exponent, the functions $M_1$, $M_2$, $f$ and $g$ are continuous functions, $(-\Delta)^s$ is the fractional laplacian operator, $||.||_X$ is a norm in the fractional Hilbert Sobolev space $X(\Omega)$, $F(x,v(x)) = \int_0^{v(x)} f(\tau)d\tau$, $G(x,u(x)) = \int_0^{u(x)} g(\tau)d\tau$, $r_1$ and $r_2$ are positive constants, $\lambda$ and $\gamma$ are real parameters. For this problem we prove the existence of infinitely many solutions, via a suitable truncation argument and exploring the genus theory introduced by Krasnoselskii. Also we show that these solutions are sufficiently regular and solve the problem pointwise.

*Mathematics subject classification* (2010): 35J60, 35J70, 58E05.
*Keywords and phrases*: Kirchhoff-type equations, fractional laplacian operator, Krasnoselskii's genus, critical growth, regularity.

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