

ON UNBOUNDED OSCILLATION OF FOURTH ORDER FUNCTIONAL DIFFERENCE EQUATIONS

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Abstract. In this work, an illustrative discussion have been made on unbounded oscillation properties of a class of fourth order neutral functional difference equations of the form:

$$\Delta^2(r(n)\Delta^2(y(n) + p(n)y(n - \tau))) + g(n)G(y(n - \sigma)) - h(n)H(y(n - \alpha)) = 0$$

under the assumptions

$$\sum_{n=0}^{\infty} \frac{n}{r(n)} = \infty, \quad \sum_{n=0}^{\infty} \frac{n}{r(n)} < \infty.$$

New oscillation criteria have been established for different ranges of $p(n)$ with $|p(n)| < \infty$.

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