

EXISTENCE RESULTS AND CONTINUITY DEPENDENCE OF SOLUTIONS FOR FRACTIONAL EQUATIONS

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Abstract. Using two fractional-order integral inequalities we investigate the existence and uniqueness of solutions of the fractional nonlinear Volterra integral equation and the fractional nonlinear integrodifferential equation in Banach space C_{ξ} , using an adequate norm, $\|\cdot\|_{\xi, \infty}$. Besides, we study the solution estimate and investigate their continuous dependence.

Mathematics subject classification (2010): 26A33, 34A08, 34A12, 34A60, 34G20.

Keywords and phrases: Fractional differential equations, fractional integrodifferential, existence and uniqueness, estimate and continuous dependence.

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