

UNIQUE SOLVABILITY OF FRACTIONAL QUADRATIC NONLINEAR INTEGRAL EQUATIONS

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Abstract. In this paper we study the existence of monotonic solutions of fractional nonlinear quadratic integral equations in the space of Lebesgue integrable functions on $[0, \tau]$. The uniqueness of the solution is also discussed. In addition an example is given to illustrate our abstract results.

Mathematics subject classification (2010): 45G10, 47H30, 47N20.

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REFERENCES

- [1] A. ABDELDAIM, *On some new Gronwall-Bellman-Ou-Iang type integral inequalities to study certain epidemic models*, J. Integral Equations Appl., **24**, (2012), 149–166.
- [2] J. APPELL AND P. P. ZABREJKO, *Nonlinear Superposition Operators*, Cambridge Tracts in Mathematics **95**, Cambridge University Press, Cambridge, 1990.
- [3] N. T. J. BAILEY, *The Mathematical Theory of Infectious Diseases and Its Applications*, Hafner Press, New York, 1975.
- [4] J. BANAŚ AND K. GOEBEL, *Measures of Noncompactness in Banach Spaces*, Lect. Notes in Math. **60**, M. Dekker, New York-Basel, 1980.
- [5] J. BANAŚ AND Z. KNAP, *Measures of weak noncompactness and nonlinear integral equations of convolution type*, J. Math. Anal. Appl., **146**, (1990), 353–362.
- [6] A. BELLOUR, M. BOUSSELSAL AND M. AZIZ, *Integrable solutions of a nonlinear integral equation related to some epidemic models*, Glasnik Matematički, **49**, (2014), 395–406.
- [7] M. CICHÓN AND M. METWALI, *On a fixed point theorem for the product of operators*, J. Fixed Point Theory Appl., **18**, (2016), 753–770.
- [8] M. CICHÓN AND M. METWALI, *Existence of monotonic L_ϕ -solutions for quadratic Volterra functional-integral equations*, Electron. J. Qual. Theory Differ. Equ., 2015, (2015), 1–16.
- [9] M. CICHÓN AND M. METWALI, *On monotonic integrable solutions for quadratic functional integral equations*, Mediterr. J. Math., **10**, (2013), 909–926.
- [10] M. CICHÓN AND M. METWALI, *On the existence of solutions for quadratic integral equations in Orlicz spaces*, Math. Slovaca, **66**, (2016), 1413–1426.
- [11] M. A. DARWISH, *Monotonic solutions of a functional integral equation of Urysohn type*, PanAmer. Math. J., **18**, 4 (2008), 17–28.
- [12] M. A. DARWISH, J. R. GRAEF AND K. SADARANGANI, *On Urysohn-Volterra Fractional Quadratic Integral Equations*, J. Appl. Anal. Comput., **8**, (2017), 331–343.
- [13] M. A. DARWISH AND J. HENDERSON, *Solvability of a functional integral equation under Carathéodory conditions*, Comm. Appl. Nonlinear Anal., **16**, 1 (2009), 23–36.
- [14] M. A. DARWISH AND K. SADARANGANI, *Nonincreasing solutions of a functional integral equation with Carathéodory perturbed*, Mediterr. J. Math., **12**, (2015), 63–76.
- [15] M. A. DARWISH, J. HENDERSON AND D. O'REGAN, *Existence and asymptotic stability of solutions of a perturbed fractional functional-integral equation with linear modification of the argument*, Bull. Korean Math. Soc., **48**, (2011), 539–553.

- [16] M. A. DARWISH AND B. SAMET, *On Erdélyi-Kober quadratic functional-integral equation in Banach algebra*, Numer. Funct. Anal. Optim., **39**, (2018), 276–294.
- [17] H. DING, M. LIU, AND J. J. NIETO, *Multiple positive solutions for quadratic integral equations of fractional order*, J. Funct. Spaces, 2017, Art. ID 4571067, 8 pp.
- [18] N. ERZAKOVA, *Compactness in measure and measure of noncompactness*, Siberian Math. J., **38**, (1997), 926–928.
- [19] G. GRIPENBERG, *On some epidemic models*, Quart. Appl. Math., **39**, (1981), 317–327.
- [20] G. GRIPENBERG, *Periodic solutions of an epidemic model*, J. Math. Biol., **10**, (1980), 271–280.
- [21] M. A. KRASNOSEL'SKII, P. P. ZABREJKO, E. I. PUSTYLNİK AND P. E. SOBOLEVSKII, *Integral Operators in Spaces of Summable Functions*, Nauka, Moscow, 1966 (English translation: Noordhoff, Leyden, 1976).
- [22] J. KRZYŻ, *On monotonicity-preserving transformations*, Ann. UMCS, **6**, (1952), 91–111.
- [23] K. S. MILLER AND B. ROSS, *An Introduction to Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons, New York 1993.
- [24] M. METWALI, *On a class of quadratic Urysohn-Hammerstein integral equations of mixed type and initial value problem of fractional order*, Mediter. J. Math., **13**, (2016), 2691–2707.
- [25] M. METWALI, *On some qualitative properties of integrable solutions for Cauchy-type problem of fractional order*, J. Math. Appl., **40**, (2017), 121–134.
- [26] M. METWALI, *Solvability of functional quadratic integral equations with perturbation*, Opuscula Math., **33**, (2013), 725–739.
- [27] I. M. OLARU, *Generalization of an integral equation related to some epidemic models*, Carpathian J. Math., **26** (2010), 92–96.
- [28] B. G. PACHPATTE, *On a new inequality suggested by the study of certain epidemic models*, J. Math. Anal. Appl., **195**, (1995), 638–644.
- [29] L. LI, F. MENG AND P. JU, *Some new integral inequalities and their applications in studying the stability of nonlinear integro-differential equations with time delay*, J. Math. Anal. Appl., **377**, (2011), 853–862.
- [30] M. VÄTH, *Volterra and Integral Equations of Vector Functions*, Marcel Dekker, New York-Basel, 2000.