

MULTIPLE SOLUTIONS TO A THIRD-ORDER THREE-POINT NONHOMOGENEOUS BOUNDARY VALUE PROBLEM AIDED BY NONLINEAR PROGRAMMING METHODS

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Abstract. In this work, we consider a third order equation of three points with non-homogeneous conditions at the border. We apply Avery Peterson's theorem, and present a theoretical result that guarantees the existence of multiple solutions to this problem under certain conditions. In addition, we present non-trivial examples and a new numerical method based on optimization is introduced.

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