Differential
Equations
& Applications
Volume 13, Number 2 (2021), 115–149
doi:10.7153/dea-2021-13-08

A COMPLETE FROBENIUS TYPE METHOD FOR LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THIRD ORDER

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Abstract. The main subject of this paper is the study of third order linear partial differential equations with analytic coefficients in a two variables domain. We aim the existence of solutions by algorithmic means, in the real or complex analytical case. This is done by introducing methods inspired by the classical method of Frobenius method for analytic second order linear ordinary differential equations. We introduce a notion of Euler type partial differential equation. To such a PDE we associate an indicial cubic, which is an affine plane curve of degree three. Points in this curve are associate to solutions of the Euler PDE. Then comes the concept of regular singularity for the PDE, followed by a notion of resonance and a partial classification of PDEs having such regular singularities. Finally, we obtain convergence theorems, which must necessarily take into account the existence of resonances and the type of PDE (parabolic, elliptical or hyperbolic). We provide some examples of PDEs that may be treated with our methods. This is the first study in this rich subject. Our results are a first step in the reintroduction of techniques from ordinary differential equations in the study of classical problems involving partial differential equations. Our solutions are constructive and computationally viable.

Mathematics subject classification (2010): 35A20, 35A24, 35A30, 35C10.
Keywords and phrases: Frobenius method, regular singularity, partial differential equation.

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