

## EXISTENCE AND BOUNDARY BEHAVIOR OF SOLUTIONS FOR BOUNDARY BLOW-UP QUASILINEAR ELLIPTIC PROBLEMS WITH GRADIENT TERMS

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*Abstract.* In this paper, by sub-supersolution methods, Karamata regular variation theory and perturbation method, we study the existence, uniqueness and asymptotic behavior of solutions near the boundary to quasilinear elliptic problem

$$\begin{cases} \operatorname{div}(|\nabla u|^{m-2}\nabla u) = b(x)f(u)(1 + |\nabla u|^{q(m-1)}), & x \in \Omega, \\ u > 0, & x \in \Omega, \\ u|_{\partial\Omega} = +\infty, \end{cases}$$

where  $\Omega$  is a bounded domain with smooth boundary in  $\mathbf{R}^N$  ( $N \geq 2$ ),  $1 < m \leq 2$ ,  $0 < q \leq m/(m-1)$ .  $b \in C^\alpha(\bar{\Omega})$  ( $\alpha \in (0, 1)$ ) is positive in  $\Omega$ , and may be vanishing on the boundary, and  $f \in C^1[0, +\infty)$ ,  $f(0) = 0$ , is increase on  $(0, +\infty)$  and normalized regularly varying at infinity with positive index  $p$  and  $p + (q-1)(m-1) > 0$ .

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