

ON PERSISTENCE AND INVADING SPECIES IN ECOLOGICAL DYNAMICS

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Abstract. The general problem of persistence of species, amounts to define interactions between them ensuring the survival of all the species initially present in the system. It appears that several relevant persistence schemes induce “forbidden sets” of zero measure for topological reasons. These peculiarities (without practical consequences) are nevertheless not consistent with certain mathematical definitions of persistence, which are too much restrictive. We come back to definitions of McGehee – Armstrong and their celebrated counter-example to the so-called “competitive exclusion principle”. We develop these concepts in relation with invasion properties of the species in a rather practical and computational framework. Several examples of communities exhibiting persistence without internal rest point (which necessarily exists according to strict persistence definitions) are given, with explicit description of the attractors, forbidden sets and invasion properties. Mechanisms of contamination of these properties (based on elementary cartesian product and structural stability) are given, showing the widespread nature of these schemes.

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