

LIAPUNOV FUNCTIONS FOR NEURAL NETWORK MODELS

MÁRTON NEOGRÁDY-KISS AND PÉTER L. SIMON*

Abstract. The dynamical behaviour of continuous time recurrent neural network models is studied with emphasis on global stability of a unique equilibrium. First we show in a unified context two Liapunov functions that were introduced in the nineties by Hopfield, Grossberg, Matsouka and Forti. Then we introduce a class of networks for which the model becomes a special cooperative system with a unique globally stable steady state. Finally, we show that periodic orbits may occur when the sufficient conditions for the existence of Liapunov functions are violated.

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