

COMPARISON THEOREMS ON THE OSCILLATION OF THIRD-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH MIXED DEVIATING ARGUMENTS IN NEUTRAL TERM

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Abstract. This study purposes to present some new comparison theorems that guarantee the oscillation of all solutions of third-order functional differential equations with mixed neutral term i.e., the neutral term contains both retarded and advanced arguments. The obtained results are based on comparisons with associated first-order delay differential inequalities and first-order delay differential equations, and they are applicable to both cases where the neutral coefficients of differential equation are unbounded and/or bounded. Illustrative examples are also provided to validate the main results.

Mathematics subject classification (2020): 34K11, 34K12, 34K40.

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