

## EXISTENCE RESULTS FOR THE $\sigma$ -HILFER HYBRID FRACTIONAL BOUNDARY VALUE PROBLEM INVOLVING A WEIGHTED $\phi$ -LAPLACIAN OPERATOR

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*Abstract.* In this paper, we are interested in the existence of positive solutions for the Hilfer hybrid fractional equation involving a weighted two dimensional  $\phi$ -Laplacian operator with the integral-infinite point boundary conditions. In this approach, we transform the given fractional differential equation into an equivalent integral equation. Then we establish sufficient conditions and employ the fixed point index arguments to obtain new results on the existence of positive solutions. Examples illustrating the main results are also constructed. This work contains several new ideas, and gives a unified approach applicable to many boundary value problems involving  $(p, q)$ -Laplacian type operators.

*Mathematics subject classification (2020):* 34B15, 34B16, 34B18.

*Keywords and phrases:* Hybrid bvp, positive solution, fixed point index.

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