Differential
Equations
& Applications
Volume 14, Number 1 (2022), 83–97
doi:10.7153/dea-2022-14-06

EXISTENCE RESULT FOR A PROBLEM INVOLVING ψ–RIEMANN–LIOUVILLE FRACTIONAL DERIVATIVE ON UNBOUNDED DOMAIN

KHEIREDDINE BENIA, MOUSTAFA BEDDANI, MICHAL FEČKAN AND BENAOUDA HEDIA*

Abstract. This paper deals with the the existence of solution sets and its topological structure for a fractional differential equation with ψ-Riemann-Liouville fractional derivative on $(0,\infty)$ in a special Banach space. Our approach is based on a fixed point theorem for Meir-Keeler condensing operators combined with measure of non-compactness. An example is given to illustrate our approach.

Mathematics subject classification (2020): 26A33, 34A08, 47H08, 47H10.
Keywords and phrases: Measure of non-compactness, Meir-Keeler condensing operators, special Banach space, fixed point theorem, ψ-Riemann-Liouville fractional derivative.

REFERENCES

[1] S. ABBAS, M. BENCHOHRA, J. GRAEF AND J. HENDERSON, *Implicit Fractional Differential and Integral Equations; Existence and Stability*, De Gruyter, Berlin, 2018.
[2] S. ABBAS, M. BENCHOHRA AND G. M. N'GUÉRÉKATA, *Topics in Fractional Differential Equations*, Springer, New York, 2012.
[3] S. ABBAS, M. BENCHOHRA AND G. M. N'GUÉRÉKATA, *Advanced Fractional Differential and Integral Equations*, Nova Science Publishers, New York, 2015.
[4] A. AGHAJANI, M. MURSALEEN AND A. SHOLE HAGHIGHI, *Fixed point theorems for Meir-Keeler condensing operators via measure of noncompactness*, Acta Math. Sci. Ser. B (Engl. Ed.) **35** (3).
[5] B. AHMAD, *Existence of solutions for irregular boundary value problems of nonlinear fractional differential equations*, Appl. Math. Lett. **23** (2010), 390–394.
[6] A. ARARA, M. BENCHOHRA, N. HAMIDI, J. J. NIETO, *Fractional order differential equations on an unbounded domain*, Nonlinear Anal. TMA **72** (2010), 580–586.
[7] K. BALACHANDRAN, S. KIRUTHIKA, J. J. TRUJILLO, *Existence results for fractional impulsive integrodifferential equations in Banach spaces*, Commun. Nonlinear Sci. Numer. Simul. **16** (2011), 1970–1977.
[8] K. BALACHANDRAN, J. Y. PARK, *Nonlocal Cauchy problem for abstract fractional semilinear evolution equations*, Nonlinear Anal. TMA **71** (2009), 4471–4475.
[9] K. BALACHANDRAN, J. J. TRUJILLO, *The nonlocal Cauchy problem for nonlinear fractional integrodifferential equations in Banach spaces*, Nonlinear Anal. TMA **72** (2010), 4587–4593.
[10] J. BANAŚ, K. GOEBEL, *Measures of noncompactness in Banach spaces*, Lecture Note in Pure App. Math. **60**, Dekker, New York, 1980.
[11] J. BANAŚ, M. MURSALEEN, *Sequence spaces and measures of noncompactness with applications to differential and integral equations*, Springer, New Delhi, 2014.
[12] M. BEDDANI AND B. HEDIA, *Solution sets for fractional differential inclusions*, J. Fractional Calc. Appl. **10** (2) July (2019) 273–289.
[13] M. BEDDANI AND B. HEDIA, *Existence result for a fractional differential equation involvinga special derivative*, Moroccan J. of Pure and Appl. Anal. **8(1)** (2022) 67–77.

Paper DEA-14-06

[14] M. BEDDANI AND B. HEDIA, *Existence result for fractional differential equation on unbounded domain*, Kragujev. J. Math **48** (5), (2024) 755–766.
[15] F. Z. BERRABAH, B. HEDIA AND J. HENDERSON, *Fully Hadamard and Erdélyi-Kober-type integral boundary value problem of a coupled system of implicit differential equations*, Turk. J. Math. **43** (2019), 1308–1329.
[16] C. DERBAZI, Z. BAITICHE, M. BENCHOHRA, *Cauchy problem with ψ-Caputo fractional derivative in Banach spaces*, Advances in the Theory of Nonlinear Analysis and its Applications **4** (2020), 349–361.
[17] D. J. GUO, V. LAKSHMIKANTHAM, X. LIU, *Nonlinear Integral Equations in Abstract Spaces*, Kluwer Academic Publishers, Dordrecht, 1996.
[18] B. HEDIA, *Non-local Conditions for Semi-linear Fractional Differential Equations with Hilfer Derivative*, Springer proceeding in mathematics and statistics 303, ICFDA 2018, Amman, Jordan, July 16–18, 69–83.
[19] A. G. IBRAHIM, N. A. AL SARORI, *Mild Solutions for Nonlocal Impulsive Fractional Semilinear Differential Inclusions with Delay in Banach Spaces*, Applied Mathematics **4** (2013), 40–56.
[20] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and Applications of Fractional Differential Equations*, Elsevier B. V. Amsterdam, 2006.
[21] K. D. KUCCHE, A. D. MALI, *On the Nonlinear ψ-Hilfer Hybrid Fractional Differential Equations*, Mathematics, (2020) arXiv:2008.06306.
[22] D. LUO, J. R. WANG, M. FECKAN, *Applying fractional calculus to analyze economic growth modelling*, J. Appl. Math. Stat. Inform. **14** (2018), 25–36.
[23] G. M. N'GUEREKATA, *A Cauchy problem for some fractional abstract differential equation with non local conditions*, Nonlinear Anal. TMA **70** (2009), 1873–1876.
[24] I. PODLUBNY, *Fractional Differential Equations*, in: Mathematics in Science and Engineering, vol. **198**, Academic Press, New York, London, Toronto, 1999.
[25] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Yverdon, 1993.
[26] H. A. H. SALEM, *On the fractional calculus in abstract spaces and their applications to the Dirichlet-type problem of fractional order*, Comput. Math. Appl. **59** (2010), 1278–1293.
[27] H. A. H. SALEM, *Multi-term fractional differential equation in reflexive Banach space*, Math. Comput. Modelling. **49** (2009), 829–834.
[28] A. R. SEADAWY, D. LU AND M. M. A. KHATER, *New wave solutions for the fractional-order biological population model, time fractional burgers, Drinfeld-Sokolov-Wilson and system of shallow water wave equations and their applications*, European Journal of Computational Mechanics, **26** (2017), 508–524.
[29] X. SU, *Solutions to boundary value problem of fractional order on unbounded domains in a Banach space*, Nonlinear Analysis **74** (2011), 2844–2852.
[30] J. VANTERLER DA C. SOUSA, E. CAPELAS DE OLIVEIRA, *On the ψ-Hilfer fractional derivative*, Commun. Nonlinear Sci. Numer. Simul., **60** (2018), 72–91.