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EXTREMAL SOLUTIONS AT INFINITY FOR SYMPLECTIC SYSTEMS ON TIME SCALES I — GENERA OF CONJOINED BASES

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Abstract. In this paper we present a theory of genera of conjoined bases for symplectic dynamic systems on time scales and its connections with principal solutions at infinity and antiprincipal solutions at infinity for these systems. Among other properties we prove the existence of these extremal solutions in every genus. Our results generalize and complete the results by several authors on this subject, in particular by Došlý (2000), Šepitka and Šimon Hilscher (2016), and the author and Šimon Hilscher (2020). Some of our result are new even within the theory of genera of conjoined bases for linear Hamiltonian differential systems and symplectic difference systems, or they complete the arguments presented therein. Throughout the paper we do not assume any normality (controllability) condition on the system. This approach requires using the Moore–Penrose pseudoinverse matrices in the situations, where the inverse matrices occurred in the traditional literature. In this context we also prove a new explicit formula for the delta derivative of the Moore–Penrose pseudoinverse. This paper is a first part of the results connected with the theory of genera. The second part would naturally continue by providing a characterization of all principal solutions of (**??**) at infinity in the given genus in terms of the initial conditions and a fixed principal solution at infinity from this genus and focusing on limit properties of above mentioned special solutions and by establishing their limit comparison at infinity.

Mathematics subject classification (2020): 34N05, 34C10, 39A12, 39A21.

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