

NONTRIVIAL SOLUTIONS FOR A NONLINEAR ν 'TH ORDER ATICI-ELOE FRACTIONAL DIFFERENCE EQUATION SATISFYING DIRICHLET BOUNDARY CONDITIONS

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Abstract. For $1 < \nu \leq 2$ a real number and $T \geq 2$ a natural number, by an application of a Krasnosel'skii-Zabreiko fixed point theorem, nontrivial solutions are established for a nonlinear ν th order Atici-Eloe fractional difference equation, $\Delta^\nu u(t) + f(u(t + \nu - 1)) = 0$, $t \in \{1, 2, \dots, T + 1\}$, satisfying the Dirichlet boundary conditions $u(\nu - 2) = u(\nu + T + 1) = 0$, where $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $\lim_{|r| \rightarrow \infty} \frac{f(r)}{r}$ exists.

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