

MEASURE OF NONCOMPACTNESS AND FRACTIONAL HYBRID DIFFERENTIAL EQUATIONS WITH HYBRID CONDITIONS

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Abstract. This paper deals with the existence of solutions for hybrid fractional differential equations involving Caputo fractional derivative of order $2 < \zeta \leq 3$. We base our arguments on a generalization of Darbo's fixed point theorem combined with the approaches related with measures of noncompactness in Banach algebras. To demonstrate the argument, an illustration is provided.

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