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MULTIPLE SOLUTIONS FOR A NONLINEAR DISCRETE PROBLEM OF THE SECOND ORDER

LINGJU KONG* AND MIN WANG

Abstract. We study the existence of multiple nontrivial solutions of the second order discrete problem

$$\begin{cases} -\Delta^2 u(k-1) = f(k,u(k)),\ k \in [1,N]_{\mathbb{Z}}, \\ u(0) = 0,\ u(N+1) = \mu u(N). \end{cases}$$

Our first theorem provides criteria for the existence of at least two nontrivial solutions of the problem, and also finds conditions under which the two solutions are sign-changing. Our second theorem proves, under some appropriate assumptions, that the problem has at least three nontrivial solutions, one of which is positive, one is negative, and one is sign-changing. As applications of our theorems, we further obtain several existence results for an associated eigenvalue problem. We include two examples in the paper to show the applicability of our results. Our theorems are proved by employing variational approaches, combined with the classic mountain pass lemma and a result on the invariant sets of descending flow.

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