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QUALITATIVE ANALYSIS OF DYNAMIC EQUATIONS ON TIME SCALES USING LYAPUNOV FUNCTIONS

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Abstract. We employ Lyapunov functions to study boundedness and stability of dynamic equations on time scales. Most of our Lyapunov functions involve the term $|x|$ and its Δ-derivative. In particular, we prove general theorems regarding qualitative analysis of solutions of delay dynamical systems and then use Lyapunov functionals that partially include $|x|$ to provide examples.

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