

P-PERIODIC SOLUTIONS OF A *q*-INTEGRAL EQUATION WITH FINITE DELAY

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Abstract. A Volterra type integral equation with a finite delay is considered on a discrete non-additive time scale domain $q^{\mathbb{N}_0} = \{q^n : n \in \mathbb{N}_0\}$, where $k \in \mathbb{N}$, $q > 1$. The existence of periodic solutions of this equation, which we call a *q*-integral equation, are shown employing the contraction mapping principle and a fixed point theorem due to Krasnosel'skii.

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