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p–DEFORMATION

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Abstract. In this article we introduce the so called p-deformed algebra $\mathscr{V}$. The notion of p-deformation is connected to the well-known q-deformation by the simple relation $p = \frac{q^2+q^{-2}}{2}$. Thus the p-deformed algebra $\mathscr{V}$ will have representations in terms of q-difference operators. There are isomorphisms of $\mathscr{V}$ to the q-deformed Weyl algebra $\mathscr{W}$ and to the well known algebra $\mathscr{U} = \mathscr{U}_q$, the q-deformation of the universal enveloping algebra $U_q(\mathfrak{sl}(2))$, extended by an involution. It turns out that the presentation of the p-deformed algebra $\mathscr{V}$ is more symmetric than the ones of its q counterparts. Especially the limit $p \to \pm 1$ can be performed in a direct and quite consistent manner. For $p^2 = 1$ the p-deformed algebra contains copies of the classical Weyl algebra, the Lie superalgebra $\mathfrak{osp}(1|2)$ and the Lie algebra $\mathfrak{sl}(2)$. Finally we will see that the p-deformed algebra $\mathscr{V}$ contains a "squared copy" of itself.

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