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STABILITY OF SOLUTIONS TO ABSTRACT EVOLUTION EQUATIONS IN BANACH SPACES UNDER NONCLASSICAL ASSUMPTIONS

NGUYEN S. HOANG

Abstract. The stability of the solution to the equation $(*)\dot{u} = F(t,u) + f(t)$, $t \geqslant 0$, $u(0) = u_0$ is studied. Here $F(t,u)$ is a nonlinear operator in a Banach space $\mathscr{X}$ for any fixed $t \geqslant 0$ and $F(t,0) = 0$, $\forall t \geqslant 0$. We assume that the Fréchet derivative of $F(t,u)$ is Hölder continuous of order $q > 0$ with respect to u for any fixed $t \geqslant 0$, i.e., $\|F'_u(t,w) - F'_u(t,v)\| \leqslant \alpha(t)\|v - w\|^q$, $q > 0$. We proved that the equilibrium solution $v = 0$ to the equation $\dot{v} = F(t,v)$ is Lyapunov stable under persistently acting perturbation $f(t)$ if $\sup_{t\geqslant 0}\int_0^t \alpha(\xi)\|U(t,\xi)\|\,d\xi < \infty$ and $\sup_{t\geqslant 0}\|U(t)\| < \infty$. Here, $U(t) := U(t,0)$ and $U(t,\xi)$ is the solution to the equation $\frac{d}{dt}U(t,\xi) = F'_u(t,0)U(t,\xi)$, $t \geqslant \xi$, $U(\xi,\xi) = I$, where I is the identity operator in $\mathscr{X}$. Sufficient conditions for the solution $u(t)$ to equation (*) to be bounded and for $\lim_{t\to\infty} u(t) = 0$ are proposed and justified. Stability of solutions to equations with unbounded operators in Hilbert spaces is also studied.

Mathematics subject classification (2020): 34G20, 37L05, 44J05, 47J35.
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