

EXPONENTIAL AND HYERS—ULAM STABILITY OF IMPULSIVE LINEAR SYSTEM OF FIRST ORDER

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Abstract. In this manuscript, we study the exponential stability and Hyers–Ulam stability of the linear first order impulsive differential system. We prove that the homogeneous impulsive system is exponentially stable if and only if the solution of the corresponding non-homogeneous impulsive system is bounded. Moreover, we prove that the system is Hyers–Ulam stable if and only if it is uniformly exponentially dichotomic. We obtain our results by using the spectral decomposition theorem. To illustrate our theoretical results, at the end we give an example.

Mathematics subject classification (2020): 34D20, 34D09, 34K20, 34A37.

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