

## INITIAL BOUNDARY VALUE PROBLEM FOR A TIME FRACTIONAL WAVE EQUATION ON A METRIC GRAPH

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*Abstract.* This work devoted to IBVP problem for a time-fractional differential equation on the regular metric tree graph. Using the method of separation of variables we find exact solution of the investigated problem in the form of Fourier series. Special case for these problem are discussed, moreover in this case eigenvalues and corresponding eigenfunctions are found exactly. Sufficient classes of given functions, which provides an existence and uniqueness of solution of the considered problem, are defined. Using a-priori estimates for the solution, uniqueness of solution is proved.

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