

SPACE-TIME ANALYTIC SMOOTHING EFFECT FOR THE NONLINEAR SCHRÖDINGER EQUATIONS WITH NONLINEARITY OF EXPONENTIAL TYPE

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Abstract. In this paper, we consider the global Cauchy problem for the nonlinear Schrödinger equations with nonlinearity of exponential type in higher space dimensions $n \geq 2$. In particular, we study the global existence of the solutions to the Cauchy problem with small data in the framework of intersection of Sobolev and weighted Lebesgue space: $H^{n/2} \cap \mathcal{F}H^{n/2}$. More precisely, we show that if data decay exponentially in $H^{n/2} \cap \mathcal{F}H^{n/2}$ then for any time $t \neq 0$, solutions are real-analytic in both space and time variables and have analytic continuation.

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