

## THE $e$ -POSITIVE MILD SOLUTIONS FOR IMPULSIVE EVOLUTION FRACTIONAL DIFFERENTIAL EQUATIONS WITH SECTORIAL OPERATOR

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*Abstract.* In this paper, we investigate the existence of global  $e$ -positive mild solutions to the initial value problem for a nonlinear impulsive fractional evolution differential equation involving the theory of sectorial operators. To obtain the result, we used Kuratowski's non-compactness measure theory, the Cauchy criterion and the Gronwall inequality.

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### REFERENCES

- [1] A. BABAEIA, B. P. MOGHADDAM, S. BANIHASHEMIA AND J. A. T. MACHADO, *Numerical solution of variable-order fractional integro-partial differential equations via Sinc collocation method based on single and double exponential transformations*, Commun. Nonlinear Sci. Numer. Simul. 82 (2020): 104985.
- [2] D. D. BAINOV AND A. DISHLIEV, *Population dynamics control in regard to minimizing the time necessary for the regeneration of a biomass taken away from the population*, Comp. Rend. Bulg. Sci., 42 (1989): 29–32.
- [3] D. D. BAINOV AND P. S. SIMENOV, *Systems with Impulse Effect Stability Theory and Applications*, Ellis Horwood Limited, Chichester, (1989).
- [4] I. BENEDETTI, V. OBUKHOVSKII AND V. TADDEI, *Evolution fractional differential problems with impulses and nonlocal conditions*, Disc. Cont. Dyn. Sys.-S. 13 (7) (2020): 1899.
- [5] P. CHEN, X. ZHANG AND Y. LI, *Existence and approximate controllability of fractional evolution equations with nonlocal conditions via resolvent operators*, Frac. Calc. Appl. Anal. 23.1 (2020): 268–291.
- [6] P. CHEN, X. ZHANG AND Y. LI, *Fractional non-autonomous evolution equation with nonlocal conditions*, J. Pseudo-Differ. Oper. Appl. 10.4 (2019): 955–973.
- [7] X.-L. DING AND B. AHMAD, *Analytical solutions to fractional evolution equations with almost sectorial operators*, Adv. Difference Equ. 2016.1 (2016): 1–25.
- [8] S. D. EIDELMAN AND A. N. KOCHUBEI, *Cauchy problem for fractional diffusion equations*, J. Diff. Equ. 199 (2) (2004): 211–255.
- [9] M. FEČKAN, JINRONG WANG AND Y. ZHOU, *On the new concept of solutions and existence results for impulsive fractional evolution equations*, Dyn. Partial Diff. Equ. 8.4 (2011): 345–361.
- [10] E. HERNÁNDEZ, DONAL O'REGAN AND K. BALACHANDRAN, *Existence results for abstract fractional differential equations with nonlocal conditions via resolvent operators*, Indagationes Mathematicae 24.1 (2013): 68–82.
- [11] E. HERNÁNDEZ, DONAL O'REGAN AND K. BALACHANDRAN, *On recent developments in the theory of abstract differential equations with fractional derivatives*, Nonlinear Analysis: Theory, Methods & Applications 73.10 (2010): 3462–3471.

- [12] M. HORANI, M. FABRIZIO, A. FAVINI AND H. TANABE, *Fractional Cauchy problems and applications*, Disc. Cont. Dyn. Sys.-S. 13.8 (2020): 2259.
- [13] L. N. HUYNH, Y. ZHOU, D. O'REGAN AND N. H. TUAN, *Fractional Landweber method for an initial inverse problem for time-fractional wave equations*, Applicable Anal. (2019): 1–19.
- [14] G. JIANG, Q. LU, *Impulsive state feedback control of a predator-prey model*, J. Comput. Appl. Math., 200 (2007): 193–207.
- [15] C. LIZAMA AND GASTON M. N'GUÉRÉKATA, *Mild solutions for abstract fractional differential equations*, Applicable Anal. 92.8 (2013): 1731–1754.
- [16] J. A. T. MACHADO AND A. M. LOPES, *Fractional-order kinematic analysis of biomechanical inspired manipulators*, J. Vibration Control 26.1-2 (2020): 102–111.
- [17] S. NENOV, *Impulsive controllability and optimization problems in population dynamics*, Nonl. Anal., 36 (1999), 881–890.
- [18] MANUEL D. ORTIGUEIRA AND J. A. T. MACHADO, *On the properties of some operators under the perspective of fractional system theory*, Commun. Nonlinear Sci. Numer. Simul. 82 (2020): 105022.
- [19] J. VANTERLER DA C. SOUSA, L. A. MAGNA AND E. CAPELAS DE OLIVEIRA, *Fractional calculus and the ESR test*, AIMS Math. 2.4 (2017): 692–705.
- [20] J. VANTERLER DA C. SOUSA, L. A. MAGNA, MAGUN N. N. DOS SANTOS AND E. CAPELAS DE OLIVEIRA, *Validation of a fractional model for erythrocyte sedimentation rate*, Comput. Appl. Math. 37.5 (2018): 6903–6919.
- [21] J. VANTERLER DA C. SOUSA, T. ABDELJAWAD AND D. S. OLIVEIRA, *Mild and classical solutions for fractional evolution differential equation*, Palestine J. Math. 11 (2) (2022): 229–242.
- [22] J. VANTERLER DA C. SOUSA, D. S. OLIVEIRA, AND E. CAPELAS DE OLIVEIRA, *A note on the mild solutions of Hilfer impulsive fractional differential equations*, Chaos, Solitons & Fractals. 147 (2021): 110944.
- [23] J. VANTERLER DA C. SOUSA, FAHD JARAD, AND THABET ABDELJAWAD, *Existence of mild solutions to Hilfer fractional evolution equations in Banach space*, Annals Funct. Anal. 12.1 (2021): 1–16.
- [24] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *On the  $\Psi$ -fractional integral and applications*, Comput. Appl. Math. 38.1 (2019): 4.
- [25] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *Fractional order pseudoparabolic partial differential equation: Ulam-Hyers stability*, Bull. Braz. Math. Soc., New Series 50.2 (2019): 481–496.
- [26] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *A Gronwall inequality and the Cauchy-type problem by means of  $\psi$ -Hilfer operator*, Diff. Equ. Appl. 11 (2019): 87–106.
- [27] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *Leibniz type rule:  $\psi$ -Hilfer fractional operator*, Commun. Nonlinear Sci. Numer. Simul. 77 (2019): 305–311.
- [28] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *Ulam-Hyers stability of a nonlinear fractional Volterra integro-differential equation*, Appl. Math. Lett. 81 (2018): 50–56.
- [29] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *On the Ulam-Hyers-Rassias stability for nonlinear fractional differential equations using the  $\psi$ -Hilfer operator*, J. Fixed Point Theory Appl. 20.3 (2018): 96.
- [30] J. VANTERLER DA C. SOUSA, KISHOR D. KUCHE AND E. CAPELAS DE OLIVEIRA, *Stability of  $\psi$ -Hilfer impulsive fractional differential equations*, Appl. Math. Lett. 88 (2019): 73–80.
- [31] J. VANTERLER DA C. SOUSA AND E. CAPELAS DE OLIVEIRA, *On the  $\psi$ -Hilfer fractional derivative*, Commun. Nonlinear Sci. Numer. Simul. 60 (2018): 72–91.
- [32] R. SUBASHINI, C. RAVICHANDRAN, K. JOTHIMANI AND H. M. BASKONUS, *Existence results of Hilfer integro-differential equations with fractional order*, Disc. Cont. Dyn. Sys.-S, 13.3 (2020): 911–923.
- [33] B. SUNDARAVADIVOO, *Controllability analysis of nonlinear fractional order differential systems with state delay and non-instantaneous impulsive effects*, Disc. Cont. Dyn. Sys.-S 13.9 (2020): 2561.
- [34] X.-B. SHU AND Q. WANG, *The existence and uniqueness of mild solutions for fractional differential equations with nonlocal conditions of order  $1 < \alpha < 2$* , Comput. Math. Appl. 64.6 (2012): 2100–2110.
- [35] X.-B. SHU, L. SHU AND F. XU, *A new study on the mild solution for impulsive fractional evolution equations*, arXiv:1907.03088 (2019).

- [36] X. WANG AND X. SHU, *The existence of positive mild solutions for fractional differential evolution equations with nonlocal conditions of order  $1 < \alpha < 2$* , Adv. Difference Equ. 2015.1 (2015): 159.
- [37] JINRONG WANG, M. FEČKAN AND Y. ZHOU, *Relaxed controls for nonlinear fractional impulsive evolution equations*, J. Opt. Theory Appl. 156.1 (2013): 13–32.
- [38] JINRONG WANG, Y. ZHOU AND M. FEČKAN, *On recent developments in the theory of boundary value problems for impulsive fractional differential equations*, Comput. Math. Appl. 64.10 (2012): 3008–3020.
- [39] JINRONG WANG, Y. ZHOU AND M. FEČKAN, *Abstract Cauchy problem for fractional differential equations*, Nonlinear Dyn. 71.4 (2013): 685–700.
- [40] R.-N. WANG, D.-H. CHEN AND T.-J. XIAO, *Abstract fractional Cauchy problems with almost sectorial operators*, J. Diff. Equ. 252.1 (2012): 202–235.
- [41] Y. SHI, YA JING, *A study on the mild solution of impulsive fractional evolution equations*, Appl. Math. Comput. 273 (2016): 465–476.
- [42] H. YANG, *Positive solutions for the initial value problems of impulsive evolution equations*, J. Math. Research & Exposition. 31 (6) (2011): 1047–1056.
- [43] H. YANG AND Y. LIANG, *Positive solutions for the initial value problem of fractional evolution equations*, Abs. Appl. Anal. Vol. 2013. Hindawi, 2013.
- [44] D. ZHAO, Y. LIU AND X. LI, *Controllability for a class of semilinear fractional evolution systems via resolvent operators*, Commun. Pure Appl. Anal. 18.1 (2019): 455.
- [45] D. ZHANG AND Y. LIANG, *Existence and controllability of fractional evolution equation with sectorial operator and impulse*, Adv. Difference Equ. 2018.1 (2018): 219.