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ITERATIVE SCHEMES FOR SOLVING GENERAL VARIATIONAL INEQUALITIES

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Abstract. In this paper, we consider a new class of variational inequalities involving two operators, which is called the general variational inequality. We have shown that the general variational inequalities are equivalent to the fixed point problem using the projection technique. This equivalent fixed point formulation is used to discuss the existence of solution as well as to investigate several iterative methods for solving general variational inequalities. Some applications of the associated dynamical system coupled with finite difference are explored. Convergence analysis of the proposed methods is considered under suitable conditions. Since general variational inequalities include the variational inequalities, complementarity problems and nonlinear equations as special cases, our results continued to hold for these problems. The techniques and ideas of this paper be starting point for the future research.

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REFERENCES

[1] F. ALVAREZ, *Weak convergence of a relaxed and inertial hybrid projection-proximal point algorithm for maximal monotone operators in Hilbert space*, SIAM J. Optim., 14 (2003), 773–782.
[2] G. CRISTESCU AND L. LUPSA, *Non-Connected Convexities and Applications*, Kluwer Academic Publisher, Dordrechet, (2002).
[3] P. DANIELE, F. GIANNESSI AND A. MAUGERI, *Equilibrium Problems and Variational Models*, Kluwer Acadamic, London, (2003).
[4] R. GLOWINSKI, J. L. LIONS, AND R. TREMOLIERES, *Numerical analysis of variational inequalities*, North Holland, Amsterdam, (1981).
[5] B. S. HE, Z. H. YANG AND X. M. YUAN, *An approximate proximal-extradient type method for monotone variational inequalities*, J. Math. Anl. Appl. 300 (2) (2004), 362–374.
[6] S. JABEEN, M. A. NOOR AND K. I. NOOR, *Inertial iterative methods for general quasi variational inequalities and dynamical systems*, J. Math. Anal. 11 (3) (2020), 14–29.
[7] S. KARAMARDIAN, *Generalized complementarity problems*, J. Opt. Theory Appl. 8 (1971), 161–168.
[8] N. KIKUCHI AND J. T. ODEN, *Contact Problems in Elasticity*, SAIM Publishing Co., Philadelphia, (1988).
[9] G. M. KORPELEVICH, *The extragradient method for finding saddle points and other problems*, Ekonomika Mat. Metody, 12 (1976), 747–756.
[10] P. MARCOTTE AND J. P. DUSSAULT, *A note on a globally convergent Newton method for solving variational inequalities*, Operation Res. Letters 6 (1987), 35–42.
[11] A. NAGURNEY AND D. ZHANG, *Projected dynamical systems and variational inequalities with applications*, Kluwer Academic Publishers, Boston, Dordrecht, London 1996.
[12] M. A. NOOR, *General variational inequalities*, Appl. Math. Letters, 1 (1988), 119–121.
[13] M. A. NOOR, *Quasi variational inequalities*, Appl. Math. Letters, 1 (4) (1988), 367–370.
[14] M. A. NOOR, *New approximation schemes for general variational inequalities*, J. Math. Anal. Appl. 251 (2000), 217–230.

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[15] M. A. NOOR, *Proximal Methods for mixed variational inequalities*, J. Optim. Theory Appl. 115 (2002) 447–451.

[16] M. A. NOOR, *Some developments in general variational inequalites*, Applied Math. Comput. 152 (2004), 199–277.

[17] M. A. NOOR, *Differentiable nonconvex functions and general variational inequalities*, Applied Math. Comput. 192 (2) (2008), 623–630.

[18] M. A. NOOR, *Extended general variational inequalities*, Appl. Math. Letters, 22 (2) (2009), 182–185.

[19] M. A. NOOR, *On an implicit method for nonconvex variational inequalities*, J. Optim. Theory Appl. 147 (2010), 411–417.

[20] M. A. NOOR, *Stability of the modified projected dynamical systems*, Computers Math. Appl. 44 (2002), 1–5.

[21] M. A. NOOR AND K. I. NOOR, *New inertial approximation schemes for general quasi variational inclusions*, Filomat, 36 (18) (2022), 6071–6984.

[22] M. A. NOOR AND K. I. NOOR, *Dynamical system technique for solving quasi variational inequalities*, U.P.B. Sci. Bull., Series A, 84 (4) (2022), 55–66.

[23] M. A. NOOR AND K. I. NOOR, *Some novel aspects of quasi variational inequalities*, Earthline J. Math. Sci., 10 (1) (2022), 1–66.

[24] M. A. NOOR AND K. I. NOOR, *General biconvex functions and bivariational inequalities*, Numer. Algeb. Control Optimization, 13 (1) (2023), 11–27.

[25] M. A. NOOR, K. I. NOOR AND A. BNOUHACHEM, *On a unified implict method for variational inequalities*, J. Comput. Appl. Math. 249 (2013), 69–73.

[26] M. A. NOOR, K. I. NOOR AND A. BNOUHACHEM, *Some new iterative methods for solving variational inequalities*, Canad. J. Appl. Math. 2 (2) (2020), 1–7.

[27] M. A. NOOR, K. I. NOOR AND B. B. MOHSEN, *Some new classes of general quasi variational inequalities*, AIMS Math. 6 (6) (2021), 6406–6421.

[28] M. A. NOOR, K. I. NOOR, AND M. TH. RASSIAS, *New trends in general variational inequalities*, Acta Applicand. Mathematic, 170 (1) (2020), 981–1046.

[29] M. A. NOOR, K. I. NOOR, AND TH. M. RASSIAS, *Some aspects of variational inequalities*, J. Comput. Appl. Math. 47 (1993), 285–312.

[30] M. A. NOOR, K. I. NOOR, AND TH. M. RASSIAS, *Iterative methods for variational inequalities*, in: Differential and Integral Inequalities (Edit: Dorin Andrica and Themistocles M. Rassias), Springer Verlag, 151 (2019), 481–508.

[31] M. PATRIKSSON, *Nonlinear Programming and Variational Inequalities: A Unified Approach*, Kluwer Acadamic publishers, Drodrecht, 1998.

[32] B. T. POLYAK, *Some methods of speeding up the convergence of iterative methods*, USSR Comput. Math. Math. Phys., 4 (1964), 117.

[33] S. M. ROBINSON, *Normal maps induced by linear transformations*, Math. Oper. Res. 17 (1992), 691–714.

[34] Y. SHEHU, A. GIBALI AND S. SAGRATELLA, *Inertial projection-type method for solving quasi-variational inequalities in real Hilbert space*, J. Optim. Theory Appl. 184 (2020), 877–894.

[35] P. SHI, *Equivalence of variational inequalities with Wiener-Hopf equations*, Proc. Amer. Math. Soc. 111 (1991), 339–346.

[36] G. STAMPACCHIA, *Formes bilineaires coercitives sur les ensembles convexes*, C. R. Acad. Sci. Paris 258 (1964), 4413–4416.

[37] K. TAJI, M. FUKUSHIMA AND T. IBARAKI, *A globally convergent Newton method for solving strongly monotone variational inequalities*, Math. Program. 58 (1993), 369–383.

[38] S. ULLAH AND M. A. NOOR, *An efficient method for solving new general mixed variational inequalities*, J. Inequal. Spec. Funct. 11 (3) (2020), 1–9.

[39] Y. S. XIA AND J. WANG, *On the stability of globally projected dynamical system*, J. Optim. Theory Appl. 106 (2000), 129–150.

[40] D. L. ZU AND P. MARCOTTE, *Co-coercivity and its role in the convergence of iterative schemes for solving variational inequalities*, SIAM J. Optim. 6 (3) (1996), 714–726.