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EXISTENCE RESULTS FOR CAPUTO FRACTIONAL BOUNDARY VALUE PROBLEMS WITH UNRESTRICTED GROWTH CONDITIONS

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Abstract. This paper presents new results to fractional boundary value problems of the Caputo type with focal boundary conditions. This fractional derivative is used extensively in modelling real world applications. The main aim of this paper is to present results for the existence of solutions to ensure the usefulness in the context of modelling and providing *a priori* bounds on all possible solutions subject to a single versatile differential inequality. These results vastly expand the scope of problems which are applicable since it allows the fractional differential equation to have unrestricted growth and be nonlinear.

Mathematics subject classification (2020): 26D10, 34A98, 34A34, 34B15, 34C11.
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