

OSCILLATION CRITERIA FOR ODD-ORDER NEUTRAL DIFFERENTIAL EQUATIONS WITH DISTRIBUTED DEVIATING ARGUMENTS

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Abstract. New sufficient conditions for the oscillation of all solutions to a class of odd-order neutral differential equations with distributed deviating arguments are established. Examples illustrating the results are provided and some suggestions for further research are indicated.

Mathematics subject classification (2020): 34C10, 34K11, 34K40.

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