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BOUNDED AND UNBOUNDED POSITIVE SOLUTIONS FOR SINGULAR ϕ–LAPLACIANS COUPLED SYSTEM ON THE HALF–LINE WITH FIRST–ORDER DERIVATIVE DEPENDENCE

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Abstract. In this paper we prove by means of expansion and compression of a cone principle, the existence of a positive solution to the second order boundary value problem

$$\begin{cases} -(\phi_1(u'))'(t) = a_1(t) f_1(t,u(t),v(t),u'(t),v'(t)) \ \ t>0, \\ -(\phi_2(v'))'(t) = a_2(t) f_2(t,u(t),v(t),u'(t),v'(t)) \ \ t>0, \\ u(0)=v(0)=\lim\limits_{t\to+\infty} u'(t)=0, \lim\limits_{t\to+\infty} v'(t)=0, \end{cases}$$

where for $i=1,2$, $\phi_i\colon \mathbb{R}\to\mathbb{R}$ is an increasing homeomorphism such that $\phi_i(0)=0$, a_i is a measurable function with $a_i(t)>0$ a.e. t in some interval of $(0,+\infty)$ and the nonlinearity $f_i\colon \mathbb{R}^+\times(0,+\infty)^4\to\mathbb{R}^+$ is continuous, and may exhibit singular at $u+v=0$ and $u'+v'=0$.

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Paper DEA-15-10

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