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$\mathscr{C}$–SYMMETRIC SECOND ORDER DIFFERENTIAL OPERATORS WITH LARGE LEADING COEFFICIENT

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Abstract. We continue the spectral analysis of Sturm-Liouville operators with ccmplex coefficients. By means of asymptotic integration the Titchmarsh-Weyl m-function is determined without the nesting circle analysis. With it the resolvent is constructed. The primary case is that of a dominant leading coefficient, but Euler type cases are also considered. This leads to resolvents that are compact and even Hilbert-Schmidt.

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