

NEW UNIQUENESS CRITERION FOR CAUCHY PROBLEMS OF  
CAPUTO FRACTIONAL MULTI-TERM DIFFERENTIAL EQUATIONSYOUSSEF GHOLAMI\*, KAZEM GHANBARI, SIMA AKBARI  
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**Abstract.** The main purpose of this investigation is to revisit solvability process of the Cauchy problems of Caputo fractional two-term initial value problems. To this aim, the Green function technique has chosen to make a bridge between the operator and the fixed point theories. The appeared Green functions in this paper are constructed by the Fox-Wright functions. Our solvability tools include the existence and uniqueness criteria as novel refinements of the Banach contraction principal and Schauder fixed point theorem. This investigation will be finalized by presenting some numerical applications that illustrate proposed solvability criteria.

**Mathematics subject classification (2020):** Primary 34A08, 34A12; Secondary 33E12, 47H10.

**Keywords and phrases:** Fractional Cauchy problems, multi-term initial value problems, existence, uniqueness, Fox-Wright function, Green function.

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