

MASS EFFECT ON AN ELLIPTIC PDE INVOLVING TWO HARDY–SOBOLEV CRITICAL EXPONENTS

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Abstract. We let Ω be a bounded domain of \mathbb{R}^3 and Γ be a smooth closed curve contained in Ω . We study existence of positive solutions $u \in H_0^1(\Omega)$ to the equation

$$-\Delta u + hu = \lambda \rho_\Gamma^{-s_1} u^{5-2s_1} + \rho_\Gamma^{-s_2} u^{5-2s_2} \quad \text{in } \Omega$$

where $h : \Omega \rightarrow \mathbb{R}$ is a function and ρ_Γ is the distance function to Γ . We prove existence of solutions depending on the regular part of the Green function of linear operator. We prove the existence of positive mountain pass solutions for this Euler-Lagrange equation depending on the mass which is the regular part of the Green function of the linear operator $-\Delta + h$.

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REFERENCES

- [1] A. DIATTA AND E. H. A. THIAM, *Influence of the curvature in the existence of solutions for a two Hardy-Sobolev critical exponents*, arxiv.org/pdf/2309.04768.
- [2] O. DRUET, *Elliptic equations with critical Sobolev exponents in dimension 3*, Annales de l'Institut Henri Poincaré C, Analyse non linéaire, vol. 19, no. 2, no longer published by Elsevier, 2002.
- [3] M. M. FALL AND E. H. A. THIAM, *Hardy-Sobolev inequality with singularity a curve*, (2018): 151–181.
- [4] D. GILBARG, DAVID AND N. S. TRUDINGER, *Elliptic partial differential equations of second order*, vol. 224, no. 2, Berlin: Springer, 1977.
- [5] W. K. HAYMANN AND P. B. KENNEDY, *Subharmonic functions*, Academic Press, London, 1976.
- [6] I. E. IJAODORO AND E. H. A. THIAM, *Influence of an lp-perturbation on Hardy-Sobolev inequality with singularity a curve*, Opuscula Mathematica 41.2 (2021): 187–204.
- [7] H. JABER, *Hardy-Sobolev equations on compact Riemannian manifolds*, Nonlinear Anal. **103** (2014), 39–54.
- [8] H. JABER, *Mountain pass solutions for perturbed Hardy-Sobolev equations on compact manifolds* Analysis, 2016, vol. 36, no. 4, 287–296.
- [9] R. SCHOEN AND S. YAU, *On the proof of the positive mass conjecture in general relativity*, Comm. Math. Phys. **65** (1979), no. 1, 45–76.
- [10] R. SCHOEN AND S. YAU, *Proof of the positive action-conjecture in quantum relativity*, Phys. Rev. Lett. **42** (1979), no. 9, 547–548.