

BOUNDARY VALUE PROBLEMS FOR FRACTIONAL DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. This paper is concerned with the existence of solutions of nonlinear fractional differential inclusions with boundary conditions in a Banach space. The main result is obtained by using the set-valued analog of Mönch fixed point theorem combined with the Kuratowski measure of noncompactness.

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