

AN OPIAL-TYPE INTEGRAL INEQUALITY AND EXPONENTIALLY CONVEX FUNCTIONS

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Abstract. In this paper a certain class of convex functions in an Opial-type integral inequality is considered. Cauchy type mean value theorems are proved and used in studying Stolarsky type means defined by the observed integral inequality. Also, a method of producing n -exponentially convex and exponentially convex functions is applied. Some new Opial-type inequalities are given for different types of fractional integrals and fractional derivatives as applications.

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REFERENCES

- [1] R. P. AGARWAL AND P. Y. H. PANG, *Opial Inequalities with Applications in Differential and Difference Equations*, Kluwer Academic Publishers, Dordrecht, Boston, London (1995).
- [2] M. ANDRIĆ, J. PEČARIĆ AND I. PERIĆ, *A multiple Opial type inequality for the Riemann–Liouville fractional derivatives*, *J. Math. Inequal.*, **7** (1) (2013), 139–150.
- [3] M. ANDRIĆ, J. PEČARIĆ AND I. PERIĆ, *Composition identities for the Caputo fractional derivatives*, *Math. Inequal. Appl.*, **16** (3) (2013), 657–670.
- [4] M. ANDRIĆ, J. PEČARIĆ AND I. PERIĆ, *Improvements of composition rule for Canavati fractional derivative and applications to Opial-type inequalities*, *Dynam. Systems Appl.*, **20** (2011), 383–394.
- [5] M. ANWAR, J. JAKŠETIĆ, J. PEČARIĆ AND ATIQUUR REHMAN, *Exponential convexity, positive semi-definite matrices and fundamental inequalities*, *J. Math. Inequal.*, **4** (2) (2010), 171–189.
- [6] J. A. CANAVATI, *The Riemann–Liouville Integral*, *Nieuw Archief Voor Wiskunde* **5** (1) (1987), 53–75.
- [7] J. JAKŠETIĆ AND J. PEČARIĆ, *Exponential Convexity Method*, *J. Convex Anal.*, **20** (1) (2013), 181–197.
- [8] A. A. KILBAS, H. M. SRIVASTAVA AND J. J. TRUJILLO, *Theory and Application of Fractional Differential Equations*, North-Holland Mathematics Studies, 204, Elsevier, New York-London, 2006.
- [9] D. S. MITRINOVIĆ AND J. E. PEČARIĆ, *Generalizations of two inequalities of Godunova and Levin*, *Bull. Polish Acad. Sci. Math.*, **36** (1988), 645–648.
- [10] J. PEČARIĆ AND J. PERIĆ, *Improvements of the Giaccardi and Petrović inequality and related Stolarsky type means*, *An. Univ. Craiova Ser. Mat. Inform.*, **39** (1) (2012), 65–75.
- [11] J. E. PEČARIĆ, F. PROSCHAN AND Y. L. TONG, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, Inc. (1992).
- [12] D. V. WIDDER, *The Laplace Transform*, Princeton University Press (1941).