

NONLOCAL BOUNDARY VALUE PROBLEMS FOR HYBRID FRACTIONAL DIFFERENTIAL EQUATIONS AND INCLUSIONS OF HADAMARD TYPE

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Abstract. This paper investigates the existence of solutions for nonlocal boundary value problems of nonlinear hybrid fractional differential equations and inclusions of Hadamard type. We make use of fixed point theorems due to Dhage [7], [8] to obtain the existence results. We also discuss some examples for illustration of the main results.

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