

AN INVERSE FRACTIONAL SOURCE PROBLEM IN A SPACE OF PERIODIC SPATIAL DISTRIBUTIONS

ANDRZEJ LOPUSHANSKY, HALYNA LOPUSHANSKA AND OLGA MYAUS

Abstract. For a time fractional diffusion equation and diffusion-wave equation with Caputo partial derivatives we prove the correctness of an inverse problem. This problem is to find a solution of direct problem, which is classical in time with values in the space of periodic spatial distributions, and a source term of the equation. A time integral over-determination condition is used.

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