

## SOLVABILITY OF A LINEAR SYSTEM WITH A NONLOCAL TERM IN A BOUNDARY CONDITION

NATALIYA VASYLYEVA

*Abstract.* In this paper, we analyze a linear system for the Poisson equations with a boundary condition comprising the fractional derivative in time and the time dependent right-hand sides. A system of this type arises under studying the Muskat boundary problem with surface tension in the case of subdiffusion. First, we prove existence and uniqueness of the solution to this problem in the Hölder classes, and provide the coercive estimates of the solution. Second, we apply the obtained results together with the contraction theorem to establish the one-to-one local classical solvability to the Muskat problem governed by anomalous diffusion in the case of nonzero surface tension of a free boundary.

*Mathematics subject classification (2010):* Primary 35R35, 35C15, Secondary 35R11, 35B65.

*Keywords and phrases:* Anomalous diffusion, Muskat problem, Caputo derivative, coercive estimates.

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