

NON-EXISTENCE RESULTS AND ANALYTICAL BOUNDS OF EIGENVALUES FOR A CLASS OF FRACTIONAL EIGENVALUE PROBLEMS

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Abstract. In this paper, we study a class of linear and nonlinear fractional eigenvalue problems with fractional derivative of Caputo type. The problem is obtained by fractionalizing a term of the well-known Sturm-Liouville operator and it covers a wide class of fractional eigenvalue problems. By applying simple maximum principles, we obtain necessary conditions for the existence of eigenfunctions and analytical bounds for the eigenvalues. We also establish a uniqueness result for the nonlinear eigenvalue problem. The results in this paper are discussed in two common spaces of fractional derivatives.

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